Problems with Multiple Objectives (Multicriteria Optimization)

Min
$$f_1(x)$$
 Min $f_2(x)$ Max $f_3(x)$ Min $f_M(x)$

$$F(x) = f[f_1(x), f_2(x), ..., f_M(x)]$$

- Individual objectives are usually in contradiction with one another, hence
- If \mathbf{x}_1^* , \mathbf{x}_2^* , ..., \mathbf{x}_M^* are the solutions to individual objectives,

then $x_1^* \neq x_2^* \neq \ldots \neq x_M^*$

• If the individual objectives are controlled by different sets of variables:

$$f_{1}(\mathbf{x}) = f_{1}(\mathbf{x}_{1}) \qquad f_{2}(\mathbf{x}) = f_{2}(\mathbf{x}_{2}) \qquad f_{M}(\mathbf{x}) = f_{M}(\mathbf{x}_{M})$$
separable function
$$F(\mathbf{x}) = f_{1}(\mathbf{x}_{1}) + f_{2}(\mathbf{x}_{2}) + \dots + f_{M}(\mathbf{x}_{M}) = f_{i}(\mathbf{x}_{i})$$
where
$$\mathbf{x} = (\mathbf{x}_{1}, \mathbf{x}_{2}, \dots, \mathbf{x}_{M})$$

$$\mathbf{x}_{1} = (x_{1}, \dots, x_{t}) \qquad \mathbf{x}_{2} = (x_{t+1}, \dots, x_{s}) \qquad \mathbf{x}_{M} = (x_{q+1}, \dots, x_{n})$$

• Then, the optimum of f can be obtained by optimizing the individual f_i 's.

Min
$$f_1(x)$$
 Min $f_2(x)$ Max $f_3(x)$ Min $f_M(x)$

$$x_1^*, f_1^* \qquad x_2^*, f_2^* \qquad x_3^*, f_3^* \qquad x_M^*, f_M^*$$

$$x^* = (x_1^*, x_2^*, ..., x_M^*)$$

- All objectives are controlled by the same set of variables:
 - (a) Composite objective function:

$$f(x) = \alpha_1 f_1(x) + \alpha_2 f_2(x) + ... + \alpha_M f_M(x)$$

(b) Choose the most important to Max (Min), and put limits on the others.

Optimize
$$f(x) = f_2(x)$$

such that $f_1(x) \ge A_1$ $f_3(x) \ge A_3$... $f_M(x) \ge A_M$

(c) Optimize each of the objectives with respect to x individually to find f_i^* and the corresponding x_i^*

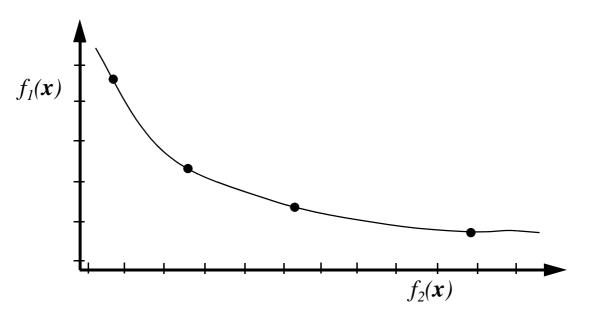
Min or Max
$$f_i$$
 f_i^*, x_i^* where $i = 1, 2, ..., M$

$$x_1^* \neq x_2^* \neq ... \neq x_M^*$$
 each design is distinct

Then,
$$Max[d_i(x)] \qquad or Min \qquad d_i^2 \qquad where \qquad d_i(x) = \frac{f_i(x) - f_i^*}{f_i^*}$$
 $i = 1$

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• A vector of design variables x^* is said to be Edgeworth - Pareto optimal if for any other design variable vector x the values of the objective functions f_i either remain the same or at least one of them worsens.



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